DIFFRACTION BY CASCADED THICK HALF PLANES COMPOSED OF PEMC METAMATERIAL

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Abstract

An analytic solution of plane wave diffraction by three parallel thick half planes composed of PEMC metamaterial is developed. Duality transformation introduced by Lindell and Sihvola is applied to transform the field produced by three semi-infinite, parallel, thick, PEC half planes to the case of three semi-infinite, parallel, thick half planes in PEMC medium. It is observed that PEC medium is the limiting case of PEMC medium. Numerical results are also produced and discussed for the effects of thickness and admittance parameters on the amplitude of the diffracted field. Numerical results are found to be in good agreement with the available numerical results on PEMC metamaterial.

Keywords: Diffraction, Duality transformation, PEMC medium, Wiener-Hopf technique, cascaded thick half planes.

1. INTRODUCTION

Diffraction of electromagnetic waves by canonical geometries such as half planes, strips, slits waveguides etc has been the subject of numerous past investigations. Lee [1,2] studied and analyzed in detail the diffraction problems related to open ended staggered plate waveguide, open ended parallel plate waveguide, bifurcated waveguide and by array of parallel plates. Usually in dealing with problems comprising of parallel plates it become cumbersome to decouple the arising Wiener-Hopf (WH) equation and thus one has to work in the domain of matrix WH equations. A comprehensive procedure for tackling the matrix version of WH equations is not yet available because it is not normally easy to split the matrix into factors being regular in appropriate half planes and these factors should have algebraic growth at infinity. The non commutativity of the factor matrices and the requirement of the radiation conditions also present further problems. Nevertheless the development and improvement of this technique is progressing steadily. For example the Wiener-Hopf Hilbert method introduced by Hurd [3] Rawlins [4] and Rawlins and Williams [5] is a powerful tool in the case when kernel matrix has only branch point singularities, while the Daniele-Kharapkov method proposed by Daniele [6] Kharapkov [7] and Jones [8] is effective for the class of matrices having only pole singularities and branch-cut singularities.

Diffraction problems related to series of parallel plates have a long and rich history. Jones [9] attempted the plane wave diffraction problem by three parallel soft semi-infinite plates which required the factorization of 3x3 matrix arose in the related WH equations. Jones tackled the problem [9] by reducing it to two WH equations which then required the factorization of a 2x2 kernel matrix instead of a 3x3 kernel matrix. Later Ibrahams [10] reconsidered the problem [9] and showed that product decomposition of involved 2x2 kernel matrix would be much simpler if Jones [9] would not have considered the system of WH equations as a fully coupled system. Asghar etal. [11] extended Jones analysis [9] from plane wave diffraction to the cases of line source and point source diffraction in still air as well as when the medium is convective. Alkumru [12] also extended Jones analysis [9] to the case of thick and impedant plates instead of three semi-infinite, parallel, plates being thin and perfectly conducting.

What has not been done is the consideration of PEMC medium for the work reported in [12]. The aim of this paper is to study the plane wave diffraction by three, semi-infinite cascaded thick half planes in a PEMC metamaterial by using the duality transformation introduced by Lindell and Sihvola [13]. Plane wave diffraction from three, semi-infinite cascaded thick half planes will help understanding the diffraction process in PEMC medium and will go a step further to complete the discussion with respect to parallel thick half planes. Since our method of solution also consists of integral transforms, the WH technique and the method of steepest descent so the notation to be used principally is that of reference [12]. In order to avoid repetition we shall omit details of calculations and shall only report necessary computational steps and major calculations essential for understanding the effects of PEMC metamaterial. Some graphs showing the effects of thickness parameter b and admittance parameter M are plotted and discussed. A good agreement with the existing literature on PEMC metamaterial is observed.

The work presented in this paper is not only mathematically important but may also have engineering applications. As polarization is always an important factor to consider in the field of transmission and reception of signals which may carry audio, video or any type of data for most of the wireless communications. Multipath effects are the common reception problems in dynamic complex electromagnetic environments. In order to reduce signal fading caused by multipath effects, diversity techniques are therefore applied to the antenna system at the receiving site [14]. Different kinds of polarization techniques have been analyzed and different pros and cons have been discussed in [15]. Hence in case of PEMC metamaterial based waveguides we can control the polarization by choosing the suitable value of parameter M and hence we can control the scattering phenomenon.

1. FORMULATION OF THE PROBLEM

The geometry of the problem is depicted in Fig. 1.

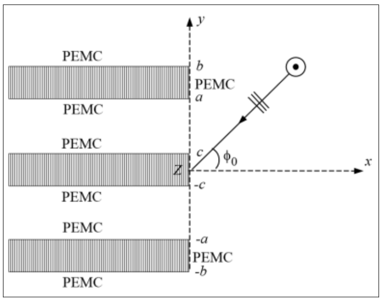

Fig. 1: Geometry of the Problem.

In semi-infinite waveguide region, scattered field is expanded into normal modes and Fourier integral transform is used elsewhere. Alkumru [12] used image bisection principle to split up the problem into even and odd excitation modes which result into a modified WH equation of second kind whose solution contains an infinite many constants satisfying an infinite number of algebraic equations for which [12] has to resort on numerical solution of this system of infinite many algebraic equations. A time factor of the form $e^{-i\omega t}$ is assumed and suppressed throughout the analysis. Let the three thick, semi-infinite, cascaded and impedance plates are located at the positions $S_1 = \{(x,y,z);\ x \in (-\infty,0), y\in(a,b); z\in(-\infty,\infty)\}$, $S_2 = \{(x,y,z);\ x \in (-\infty,0), y\in(-c,c); z\in(-\infty,\infty)\}$, and

$$S_3 = \{(x,y,z); x \in (-\infty,0), y \in (-b,-a); z \in (-\infty,\infty)\}.$$

The horizontal and vertical walls of the geometry depicted in Fig. 1 have same impedance $Z = \eta Z_0$, where Z_0 is the characteristic impedance of the free space. Diffraction of an E_z-polarized plane wave defined as

$$E_z^i = u_i(x,y) = e^{-ik(x\cos\phi_0 + y\sin\phi_0)}, \quad (1)$$

where $k = \frac{2\pi}{\lambda}$, is the free space wave number and ϕ_0 is the angle of incidence is considered by three parallel, thick, semi infinite half planes for which the even and odd excitation modes will be dealt separately in sequel. Detailed formulation can also be seen from [12]. The field reflected from the plane $y = b$, having impedance η can be written as follows:

$$u_1^r(x,y) = \frac{\eta\sin\phi_0 - 1}{\eta\sin\phi_0 + 1} e^{-ik[x\cos\phi_0 - (y-2b)\sin\phi_0]}. \quad (2)$$

Analytic solution of the problem for even excitation mode

Let us fiest consider the even excitation mode. Since the total field is symmetric at the plane $y = 0$, the normal derivative of the total electric field must vanish at $x \in (-\infty, \infty)$, $y = 0$. The Helmholtz equation satisfied by the total field $u_T^e(x,y)$ is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)u_T^e(x,y) = 0, \quad (3)$$

where the superscripts e and o denote the even and odd excitation modes. The supporting boundary conditions on the semi-infinite cascaded half planes located at $-\infty < x < 0$, $y = b$, $y = a$ and $y = c$ are

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial y}\right]u_T^e(x,b) = 0, \quad (4)$$

$$\left[1 - \frac{\eta}{ik}\frac{\partial}{\partial y}\right]u_T^e(x,a) = 0, \quad (5)$$

and

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial y}\right]u_T^e(x,c) = 0. \quad (6)$$

The boundary condition on the plane $0 < x < \infty$, $y = 0$ is

$$\frac{\partial u_T^e(x,0) = 0}{\partial y}. \quad (7)$$

The boundary condition on the wall $x = 0$, $y \in \{(0,c) \cup (a,b)\}$ is

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial x}\right]u_T^e(0,y) = 0. \quad (8)$$

In addition to the above mentioned boundary conditions the following continuity relations in the regions $0 < x < \infty$, $y = b$ and $x = 0$, $c < y < a$ should also be considered to complete the solution of the boundary value problem i.e.,

$$u_T^e(x,b^+) = u_T^e(x,b^-) \quad x > 0, \quad (9)$$

$$\frac{u_T^e(x,b^+)}{\partial y} = \frac{u_T^e(x,b^-)}{\partial y} \quad x > 0, \quad (10)$$

and

$$u_T^e(0,y^+) = u_T^e(0,y^-) \quad c < y < a, \quad (11)$$

$$\frac{\partial u_T^e(0,y^+)}{\partial x} = \frac{\partial u_T^e(0,y^+)}{\partial x} \quad c < y < a, \quad (12)$$

where the superscripts + and — denote that the corresponding limits are attained from above or below side of the respective half plane. For analysis of even mode excitation problem it is convenient to express the total field $u_T^e(x,y)$ in different regions as

$$u_T^e(x,y) = \begin{cases} u_i^e(x,y) + u_1^r(x,y) + u_{1(diff)}^e(x,y), & \text{if } -\infty < x < \infty,\ y > b; \\ u_2^e(x,y), & \text{if } c < y < a,\ x < 0; \\ u_3^e(x,y), & \text{if } 0 < y < b,\ x > 0, \end{cases}$$

(13)

where $u_j\left(j = 1,2,3\right)$ are the scattered fields in the different regions. By substituting Eqs.(1), (2) and (13) in Eqs. (3-12) we shall arrive at:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right] u_{1(diff)}^e(x,y) = 0 \quad y > b, \quad (14)$$

subject to the boundary conditions

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial y}\right] u_{1(diff)}^e(x,b) = 0 \quad x < 0, \quad (15)$$

$$\left[1 - \frac{\eta}{ik}\frac{\partial}{\partial y}\right] u_2^e(x,a) = 0 \quad x < 0, \quad (16)$$

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial y}\right] u_2^e(x,c) = 0 \quad x < 0, \quad (17)$$

$$\frac{\partial u_3^e(x,0)}{\partial y} = 0 \quad x > 0, \quad (18)$$

$$\left[1 + \frac{\eta}{ik}\frac{\partial}{\partial x}\right] u_3^e(0,y) = 0 \quad x > 0,\ y \in \{(0,c) \cup (a,b)\}. \ (19)$$

The continuity conditions shall take the form

$$u_{1(diff)}^e(x,b^+) - u_3^e(x,b^-) = -\frac{2\eta \sin\phi_0}{\eta \sin\phi_0 + 1} e^{-ik[x\cos\phi_0 + b\sin\phi_0]} \ x > 0,\ y = b,$$

(20)

$$\frac{\partial}{\partial y} u_{1(diff)}^e(x,b^+) - \frac{\partial}{\partial y} u_3^e(x,b^+) = \frac{2ik\sin\phi_0}{\eta\sin\phi_0 + 1} e^{-ik[x\cos\phi_0 + b\sin\phi_0]} \ x > 0,\ y = b,$$

(21)

$$u_2^e(0,y^+) = u_3^e(0,y^-)\ x = 0,\ c < y < a, \ (22)$$

$$\frac{\partial}{\partial x} u_2^e(0,y^+) = \frac{\partial}{\partial x} u_3^e(0,y^-)\ x = 0,\ c < y < a. (23)$$

Since $u_{1(diff)}^e(x,y)$ satisfies the Helmholtz equation in the range $x \in (-\infty,\infty)$, its Fourier transform with respect to x gives

$$\left[\frac{d^2}{dy^2} + (k^2 - \alpha^2)\right] F^e(\alpha,y) = 0, \quad (24)$$

with

$$F^e(\alpha,y) = F_-^e(\alpha,y) + F_+^e(\alpha,y), \quad (25)$$

where $F_+(\alpha,y)$ and $F_-(\alpha,y)$ are regular funtion of α in half planes, $\mathrm{Im}(\alpha) > \mathrm{Im}(k\cos\phi_0)$ and $\mathrm{Im}(\alpha) < \mathrm{Im}(k)$ respectively. The general solution of the Eq. (24) satisfying the radiation condition for $y \to \infty$ is read as

$$F_+^e(\alpha,y) + F_-^e(\alpha,y) = A^e(\alpha) e^{iK(\alpha)(y-b)}, \ (26)$$

where $K(\alpha)=\sqrt{k^2-\alpha^2}$ is the square root function defind in the complex α plane cut along $\alpha=k$ to $\alpha=k+i\infty$ and $\alpha=-k$ to $\alpha=-k-i\infty$ such that $K(0)=k$. We observe that the mathematical problem in the transformed complex plane α is same as that of Alkumru [12]. Therefore omitting the details of calculations and following the standard WH method [16] and the procedure used in [17] i.e., letting $\eta=0,$ we can obtain the results for three perfectly conducting (PEC), semi-infinte, parallel thick half planes in case of even excitation case as:

$$ik\frac{\chi_+(\alpha)}{N_+^e(\alpha)}R_+^e(\alpha)=-2k\sin\phi_0\frac{e^{-ikb\sin\phi_0}}{\alpha-\alpha_0}\frac{N_-^e(\alpha_0)}{\chi_-(\alpha_0)}-\sum_{m=1}^{\infty}\frac{K_m^e\sin(K_m^e b)}{\alpha+\alpha_m^e}\frac{N_+^e(\alpha_m^e)}{\chi_+(\alpha_m^e)}\frac{\phi_m^e}{2\alpha_m^e},$$
(27)

and in case of odd excitations as

$$ik\frac{\chi_+(\alpha)}{N_+^o(\alpha)}R_+^o(\alpha)=2k\sin\phi_0\frac{e^{-ikb\sin\phi_0}}{\alpha-\alpha_0}\frac{N_-^o(\alpha_0)}{\chi_-(0,\alpha_0)}+\sum_{m=1}^{\infty}\frac{K_m^e\cos(K_m^e b)}{\alpha+\alpha_m^o}\frac{N_+^o(\alpha_m^o)}{\chi_+(0,\alpha_m^o)}\frac{\phi_m^o}{2\alpha_m^o},$$
(28)

where $\alpha_0=k\cos\phi_0$, $\alpha=k\cos\phi$, $\alpha_m^e=\frac{2m\pi}{b}$,

$\alpha_m^o=\frac{(2m+1)\pi}{b}$, $K_m^e=\sqrt{k^2-(\alpha_m^e)^2}$,

$K_m^o=\sqrt{k^2-(\alpha_m^o)^2}$,

$\phi_m^e=f_e^m-\alpha_m^e g_m^e=\frac{2K_m^e R_+^e(\alpha_m)}{b\sin(K_m^e b)}$, and

$\phi_m^o=f_m^o-\alpha_m^o g_m^o=\frac{2K_m^o R_+^o(\alpha_m)}{b\cos(K_m^e b)}$.

Further the value of $\chi_+(\alpha)$, $\chi_-(\alpha)$, $N_+^e(\alpha)$ and $N_-^e(\alpha)$ at $\eta=0$ can be obtained from [12].

2. ANALYSIS OF THE FIELDS

The diffracted field in the region $y>b$ for the both even and odd excitations can be obtained by taking the inverse Fourier transform of $F^e(\alpha,y)$ and $F^o(\alpha,y)$ as:

$$u_{1(diff)}^e(x,y)=\frac{1}{4\pi}\int_? A^e(\alpha)e^{iK(\alpha)(y-b)-i\alpha x}d\alpha \quad (29)$$

and

$$u_{1(diff)}^o(x,y)=\frac{1}{4\pi}\int_? A^o(\alpha)e^{iK(\alpha)(y-b)-i\alpha x}d\alpha. \quad (30)$$

where ? is a straight line parallel to real axis lying in the strip $\mathrm{Im}(k\cos\phi_0)<\mathrm{Im}(\alpha)<\mathrm{Im}(k)$. To determine the far field behavior of the diffracted fields $u_{1(diff)}^{e,o}(x,y)$, we need to compute the expressions of $A^{e,o}(\alpha)$ by using Eqs. (27, 28) by bearing in mind that

$$R_+^{e,o}(\alpha)=\frac{K(\alpha)}{k\chi(\alpha)}A^{e,o}(\alpha). \quad (31)$$

Introduce the substitutions

$$x=\rho\cos\phi,\ y-b=\rho\sin\phi, \quad (32)$$

In Equations (29, 30) omitting the details of calculations, the asymptotic evaluation of the integrals in Eqs. (29, 30) by using the method of steepest decent gives the diffracted field valid for $y>b$, for three parallel, semi-infinite PEC thick half planes by taking the surface impedances η equals to zero in [12] as

$$u_{1d}(\rho,\phi)=\frac{u_{1(diff)}^{(e)}(\rho,\phi)+u_{1(diff)}^{(o)}(\rho,\phi)}{2}, \quad (33)$$

with

$$u_{1(diff)}^{(e)}(\rho,\phi)\sim\left\{u_0 D^e(\phi,\phi_0)+\frac{\sin\phi e^{i\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{N_-^e(\alpha)}{\chi_-(\alpha)}\times\sum_{m=1}^{\infty}\frac{K_m^e\sin(K_m^e b)}{2\alpha_m^e}\frac{N_+^e(\alpha_m^e)}{\chi_+(\alpha_m^e)}\frac{\phi_m^e}{\alpha_m^e-\alpha}\right\}\frac{e^{ik\rho}}{\sqrt{k\rho}}$$

(34)

and

$$u_{1(diff)}^{(o)}(\rho,\phi)\sim\left\{u_0 D^o(\phi,\phi_0)+\frac{\sin\phi e^{i\frac{3\pi}{4}}}{\sqrt{2\pi}}\frac{N_-^o(\alpha)}{\chi_-(\alpha)}\times\sum_{m=1}^{\infty}\frac{K_m^o\cos(K_m^o b)}{2\alpha_m^o}\frac{N_+^o(\alpha_m^o)}{\chi_+(\alpha_m^o)}\frac{\phi_m^o}{\alpha_m^o-\alpha}\right\}\frac{e^{ik\rho}}{\sqrt{k\rho}},$$

(35)

where $u_0=e^{-ikb\sin\phi_0}$,

$$D^e(\phi,\phi_0)=e^{-i\frac{3\pi}{4}}\sqrt{\frac{2}{\pi}}\frac{\sin\phi\sin\phi_0}{\cos\phi+\cos\phi_0}\frac{N_-^e(\alpha)N_-^e(\alpha_0)}{\chi_-(\alpha)\chi_-(\alpha_0)} \quad (36)$$

and

$$D^o(\phi,\phi_0)=e^{i\frac{3\pi}{4}}\sqrt{\frac{2}{\pi}}\frac{\sin\phi\sin\phi_0}{\cos\phi+\cos\phi_0}\frac{N_-^o(\alpha)N_-^o(\alpha_0)}{\chi_-(\alpha)\chi_-(\alpha_0)}. \quad (37)$$

We have taken $u_{1d}(\rho,\phi)=E^s$ so the amplitude of diffracted field can be written as

$$|E^s|=20\log_{10}(u_{1d}(\rho,\phi)\sqrt{k\rho}). \quad (38)$$

The diffracted field from the three parallel PEC thick half planes can be transformed to three parallel thick PEMC half planes by the concept of PEMC introduced by Lindell and Sihvola [13, 18]. It is a generalization of both PEC and PMC media. PEMC medium is defined by a scalar parameter M, known as an admittance of the surface. The PEC canonical problems have been transformed to PEMC structures by many researchers[19,20,21,22,23,24,25,26,27,28,29,30,31] through the duality transformations. Here we present an analytic scattering theory for three parallel PEMC thick half planes which is a generalization of the classical diffraction theory. The diffracted fields by PEMC interface and isotropic medium can be obtained from the known problem of PEC objects. In general, the diffracted (scattered) field has a cross-polarized component which gives nonreciprocal effect. This means that PEC and PMC are the limiting cases with no cross-polarized component. Because the PEMC medium does not allow electromagnetic energy to enter and an interface of such a medium behaves as an ideal boundary to the electromagnetic field. It has also been observed theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves but differs from the PEC and the PMC. Our interest is to transform scattered field from three parallel PEC thick half planes to three parallel PEMC thick half planes of the concerned problem by using the duality transformations [13].

The transformed fields can be obtained from the field diffracted by three parallel PEC thick half planes with the help of duality transformations as

$$\begin{pmatrix} E_d^s \\ H_d^s \end{pmatrix} = \begin{pmatrix} M\eta_0 & \eta_0 \\ \frac{-1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E^s \\ H^s \end{pmatrix}, \quad (39)$$

With $E_d^s = M\eta_0 E^s + \eta_0 H^s$ (40)

and

$$H_d^s = -\frac{1}{\eta_0} E^s + M\eta_0 H^s, \quad (41)$$

where E^s and H^s are the diffracted (scatterred) fields while E_d^s and H_d^s are the transformed fields from the three parallel PEC thick half planes and these fields satisfy the condition

$$\eta_o H_d^s = -u_z \times E_d^s. \quad (42)$$

Further the following transformation gives scattered field from the three parallel PEMC thick half planes is

$$\begin{pmatrix} E \\ H \end{pmatrix} = \frac{1}{(M\eta_0)^2 + 1} \begin{pmatrix} M\eta_0 & -\eta_0 \\ \frac{1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E_d^s \\ H_d^s \end{pmatrix}, \quad (43)$$

where E is the field diffracted (scattered) by the three parallel PEMC thick half planes can be written as

$$E = \frac{1}{(M\eta_0)^2 + 1} \left[((M\eta_0)^2 - 1)E^s - 2M\eta_0 E^s \right], \quad (44)$$

with $E^s = u_{1d}(\rho,\phi) = \frac{u_{1d}^{(e)}(\rho,\phi) + u_{1d}^{(o)}(\rho,\phi)}{2}$.

(45)

3. RESULTS AND DISCUSSION

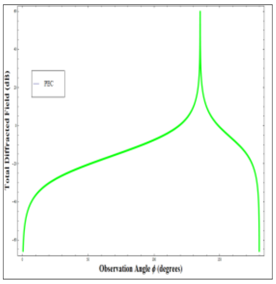

Fig. 2 (a)

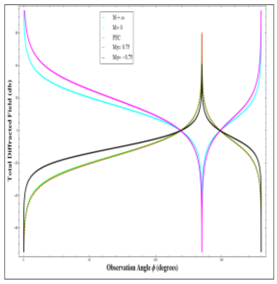
Fig. 2 (b)

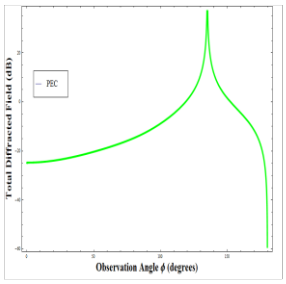

Fig. 2 (c)

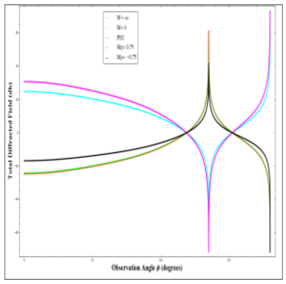
Fig. 2 (d)

Mathematica Software is employed for the numerical and graphical results; we have reproduced the results given by Lindell and Sihvola [13, 18] to get the better understanding of the behavior of cascaded PEMC thick half planes. It can be observed from the article [12] for the PEC case ($\eta = 0$), that amplitude of the diffracted field increases with increasing thickness b. An attempt is made to verify the theoretical results by the graphs. In all cases, the behavior of the diffracted field by cascaded PEC and PEMC thick half planes for thickness $b = \lambda / 4$ and $b = \lambda / 2$ against the observation angle ϕ is depicted via graphs 2 (a-d). As we can see that in the graphs, the dotted red line is overlapping the behaviour of the diffracted field at $M = \pm\infty$ with green line whereas for the value $M = 0$, the behavior corresponds to PMC case with cyan line. In the same way, the behavior of the crossed-polarized field for the values $M\eta_0 = 0.7$ shown by the magenta line while $M\eta_0 = -0.7$ represented by the black line. The PEMC medium shows maximal behavior as $M\eta_0 \to \pm 1$. The practical application of such a material is not in practice but the validation of the theoretical results available and verified. These numerical results describe the complete PEMC theory.

4. CONCLUSION

In this work, plane wave diffraction by cascaded PEMC thick half planes is investigated. The co-polarized and the cross-polarized fields depend on the parameter M. It is concluded that the both coupled electric and magnetic fields excitation can be observed analytically at the same time in the PEMC theory which leads to a most general case for the diffraction (scattering) theory. It is also incurred that the parameter M plays an important role in PEMC theory to interlink the PEC and PMC media and further through parameter M polarization in waveguides, antennas etc can be controlled which has significant applications in WLAN [14, 15]. The cross-polarized scattered fields vanish in the PEC and PMC cases and are maximal for $M\eta_0 = \pm 1$. As a check, we can see that $M = \pm\infty$, corresponds to PEC case and $M = 0$, corresponds to a PMC case.

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